A MICROMECHANICAL APPROACH TO MODELING THE EFFECT OF IMPACT INDUCED DAMAGE ON THE VIBRATION RESPONSE OF CFRP LAMINATES

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ABSTRACT

This work deals with modeling the effect of low-velocity impact damage upon the vibration response of CFRP laminates through a micro-mechanical description of the induced internal damage. The serial-parallel (SP) continuum approach is used to estimate the map of induced internal damage by considering the micro-structural interaction between the composite constituents and modifying their constitutive performance through a continuum damage formulation. An eigenvalue analysis is then done to determine the modal response of impacted laminates. The validity of the modeling approach to successfully reproduce the vibration response of impacted coupons is assessed through a comparison with an experimental test series conducted on a set of 48 CFRP laminated coupons. The results confirm the ability of the described approach in comparison to competing ones used to reproduce the experimentally observed behaviour.

1 INTRODUCTION

Composite laminates are being extensively used in advanced structural applications because of their appealing advantages when being compared to traditional structural materials. Nevertheless, these materials have also shown a susceptibility to impact damage due to lack of plastic deformation, low interlaminar strength and the laminated construction to reduce the anisotropic nature of the plies. Experimental studies consistently indicate that impact-induced damage combines three main failure modes: matrix cracking, delamination and fibre breakage, among which delamination is the most severe because it may severely degrade the stiffness and strength of the composite laminate, thus compromising the residual bearing capacity of the structural component and causing a collapse under unacceptable compressive load levels. For these reasons, over the years, a massive amount of research activity has been devoted to developing non-destructive evaluation techniques to identify this type of damage at an early stage, as well as numerical methodologies that can quantitatively predict the performance and durability of composite structures under impact events.

Among different non-destructive experimental approaches, the so-called vibration-based Non-Destructive Testing (NDT) methods have been shown to be useful tools for damage identification in laminated composites, since, in contrast to other assessment methods, these techniques provide global information, does not necessarily require that the vicinity of the damage site to be known in advance and can be employed under in-service conditions. The overall principle underlying these methods is that vibration response depends on the physical properties of the structure (mass, damping and
stiffness); therefore, changes that occur in physical properties due to damage can result in detectable variations in the response, which can serve as an indicator of structural integrity [1]. Based on this phenomenon, a considerable number of contributions in the field have appeared, e.g. [2–5]. These methods can be broadly classified into two approaches: response-based methods that use only experimental data to identify damage, and model-based methods that require numerical models. The experimental approaches have mostly focused on measuring the vibration response before and after damaging the composite, analysing the data in both the frequency and modal domains. On the other hand, the effectiveness of the model-based methods is highly dependent on the accuracy of the numerical model used. This fact is of utmost importance when applied to impact identification in composite laminates because, as stated before, induced damage is a complex mixture of multiple failure modes interacting with each other [6].

The complexity of the failure modeling in composite structures arises from the heterogeneous microstructure of composite materials. This fact represents a departure from the way that conventional materials are modeled and consequently requires unconventional approaches to dealing with them. The different entities involved, i.e. component materials, ply and laminate, span a spectra of length scales that clearly dictates the analysis approach. Over the years, due to the increased computing capabilities, research efforts have shifted from the macroscale towards the microscale. Current developments are focussed primarily on the microscale. These have proven to be suitable to accurately predict the composite mechanical performance [7–10]. This is due to the fact that failure processes of composite structures are governed by micro-structural phenomena since the failure mechanisms of composites are caused by the micro-structural complex interaction between its constituents. Obviously, this low-scale level approach does not resolve the issue of the computational cost, but presents several significant advantages when compared to the previous methods. For instance, complex behaviour when one of the constituents gets into the non-linear range can be accounted for.

Bearing this in mind, this paper addresses the modeling of the effect upon the vibration response of low-velocity impact-induced damage on CFRP laminated plates, by means of a micro-mechanical approach based on the serial/parallel mixing theory. This paper fits within the work conducted by the authors [3, 6, 11,12]. The bottom-up implementation is aimed to determine the macroscopic behaviour of the damaged heterogeneous material by understanding the micro-structural interaction between the composite constituents and modifying their constitutive performance. Damage is included within the framework through the application of Continuum Damage Mechanics (CDM) at the constituents’ materials level. Interply and fibre damage are captured within the same framework, hence does not require use of interface elements to account for interply decohesion, nor explicitly modeling each individual ply, hence resulting in a computationally less demanding simulation. The validity of the approach to successfully predict the vibration response of impacted laminates is assessed by comparing the numerical results with those obtained from an extensive experimental campaign that shows good agreement.

The following section describes the micro-mechanical approach used to determine the map of internal induced damage as well as the bottom-up scale modeling procedure to estimate the vibration behaviour of impact damaged laminates. In section 3, numerical procedures undertaken to simulate the low-velocity impact event together with the modeling of the vibration response from three different damage approaches are presented. Subsequently, numerical and experimental results are presented, compared and discussed in section 4. Finally, the conclusions of the work are presented.

2 MICRO-MECHANICAL APPROACH

Among the failure modes induced by a low-velocity impact, delamination has been found to be the most pernicious one. Experimental evidence has shown that the interply decohesion is preceded by intraply cracks that propagate along the interface of adjacent plies with different fibre orientation due to the stress concentration induced by the elastic mismatch. Within the framework of the micro-mechanical analysis, the matrix degradation that takes place due to fracture process can be addressed
by using an isotropic continuum damage formulation [13], which is based on the effective stress concept introduced by Kachanov. The damage model assumes a reduction of the effective area of the material by the formation of a homogeneous distribution of cracks. The constitutive behaviour of the damaged material can be expressed mathematically as a function of one-parameter scalar \( m d \in [0, 1] \) as:

\[
m\sigma \equiv (1 - m d) m\sigma_0 = (1 - m d) mC_0 m\epsilon
\]

where the superscripts \( m \) indicate matrix. The scalar degradation parameter \( m d \) stands for the areal density of microcracks and is used to transform the real damaged stress tensor \( m\sigma \) into an effective stress tensor \( m\sigma_0 \) corresponding to the matrix undamaged configuration, being \( mC_0 \) the constitutive tensor of the undamaged material and \( m\epsilon \) the strain tensor. The same approach can be followed to characterize fibre degradation through the scalar parameter \( f d \in [0, 1] \) as discussed below. The prediction of the internal damage parameters, as well as \( m d \) and \( f d \), and the corresponding impact-damage induced map, deserves further attention.

The fundamental problem in micro-mechanics of heterogeneous materials is estimation of the relation between the average strains in the spatially uniform component materials, i.e. \( \epsilon \) and \( \epsilon_f \), with the average or macroscopic strain. For that purpose, this work makes use of the serial-parallel (SP) continuum approach developed by Rastellini et al. [12] and Martinez et al. [11], which makes the composite behaviour dependent on both the constitutive laws of the component materials according to their volumetric fraction and to their kinematics interaction and morphological distribution inside the composite. The model is essentially based on an appropriate management of the constitutive laws of component phases within a continuum framework by making use of suitable closure equations that characterize the composite micro-mechanics kinematics. The formulation used is thoroughly described in references [11] and will not be elaborated here except for key aspects that are briefly described in the following.

The SP continuum approach was developed assuming that the components of the composite behave as parallel in the fibres alignment direction and as serial in the orthogonal direction. Consequently, it defines an iso-strain condition in the fibre (parallel) direction and an iso-stress condition in the transversal (serial) direction (see Fig. 1). The composite strain tensor is then decomposed into its parallel and serial components by using fourth-order projector tensors \( P_p \) and \( P_s \), respectively, as follows:

\[
\epsilon = \epsilon_p + \epsilon_s
\]

where

\[
\epsilon_p = P_p : \epsilon \quad \text{and} \quad \epsilon_s = P_s : \epsilon
\]

in which \( P_p = (e_1 \otimes e_1) \otimes (e_1 \otimes e_1) \), \( P_s = (I - P_p) \), \( e_1 \) the vector that defines the parallel behaviour (fibre direction), and \( I \) the identity tensor.

The compatibility or closure equations that define the stress equilibrium and establish the strain compatibility between the composite constituent materials are:

Parallel behaviour:

\[
\begin{align*}
\epsilon_p &= I_{pp} \epsilon_p = m\epsilon_p \\
\sigma_p &= I_{pp} \sigma_p + m m\epsilon_p
\end{align*}
\]

Serial behaviour:

\[
\begin{align*}
\epsilon_s &= I_{ss} \epsilon_s + m m\epsilon_s \\
\sigma_s &= I_{ss} \sigma_s = m\epsilon_s
\end{align*}
\]

where superscripts \( c, f \) and \( m \) stands for composite, fibre and matrix, respectively, and \( f \) is the volume-fraction coefficient of each composite constituent. As observed, \( \epsilon_p \) and \( \sigma_p \) are obtained straightforward from the composite strains. However, the compatibility equations are not easily

\[
\begin{align*}
\epsilon_p &= I_{pp} \epsilon_p = m\epsilon_p \\
\sigma_p &= I_{pp} \sigma_p + m m\epsilon_p
\end{align*}
\]

where superscripts \( c, f \) and \( m \) stands for composite, fibre and matrix, respectively, and \( f \) is the volume-fraction coefficient of each composite constituent. As observed, \( \epsilon_p \) and \( \sigma_p \) are obtained straightforward from the composite strains. However, the compatibility equations are not easily
fulfilled in the transversal direction, for which it is necessary to make an initial prediction on the fibre or matrix serial strain. Once the strains tensors of both components are determined, their stress tensors are calculated using a given constitutive law. It is possible to simulate the composite constituents with a non-linear behaviour law, such as damage or plasticity, as has been proved in previously works [11].

Each fibre is considered to be homogeneous, transversely isotropic, with a linearly elastic behaviour. Fibre breakage is assumed to be the only ultimate failure mode, whereas the matrix is assumed to be homogeneous, isotropic and viscoplastic. The onset of matrix damage (i.e. \( m_d > 0 \)) can be predicted by the norm of the principal stresses tensor (or other classical scalar threshold stress function) as:

\[
F(m\sigma) = \left[ \frac{\sum_{i=1}^{3} (m\sigma_i) m\sigma_i}{\sum_{i=1}^{3} m\sigma_i} \right] + \left( 1 - \frac{\sum_{i=1}^{3} (m\sigma_i)}{\sum_{i=1}^{3} m\sigma_i} \right) ||m\sigma||
\]

(6)

where the first term is a weighting function that accounts for a different degradation path for tensile and compressive stresses, with \( m\sigma_i \) and \( m\sigma_i \) being the tensile and the compressive matrix ultimate strengths, respectively. \(< >\) stands for the Macaulay brackets defined as \( <x> = 0.5 \) (\( x + |x| \)).

The variation of the damage parameter \( m_d \) is obtained by using an exponential strain-softening law [13] expressed as:

\[
m_d = 1 - \frac{m\sigma_{max}}{F(m\sigma_0)} A \left( 1 - \frac{F(m\sigma_0)}{m\sigma_{max}} \right)
\]

(7)

where \( m\sigma_{max} \) denotes the damage threshold defined as a material property, \( F(m\sigma_0) \) is the yield function and the parameter \( A \) depends upon the material fracture energy as follows:

\[
A = \frac{1}{m\gamma F(m\sigma_{max})^2 - \frac{1}{2}}
\]

(8)

in which \( m\gamma \) is the matrix elastic stiffness, \( m\gamma \) the fracture energy and \( l_f \) the fracture length.

When impact damage occurs, the weakening of the matrix due to excessive shear stresses result in the inability to develop either longitudinal, transverse or shear stresses. According to the above compatibility equations, when the matrix reaches a critical stress state its strength and stiffness is reduced, and accordingly the whole composite performance is also reduced in the transversal direction. Given that shear stresses are defined through the serial condition, the composite shear strength and stiffness are reduced as happens in a delaminated composite. One significant advantage is that intraply and interply damages are captured within the same framework.

This approach can be roughly interpreted as a phenomenological homogenisation. It has proven to be capable of successfully simulate the composite structural performance when applied to a low-velocity impact problem, especially to predict the phenomenon of initiation and propagation of interlaminar damage [6]. This allows the estimation of the impact-damage induced map and thus the corresponding internal damage parameters \( \gamma d \) and \( m d \) required to subsequently investigate the effect of impact-induced damage on the vibration response.

Assuming the lamina to be macroscopically homogeneous by focusing on the average response, the effective engineering constants of the lamina reinforced with continuous unidirectional fibres aligned in the 1 direction, are calculated in terms of the elastic properties of the constituent materials and their concentrations through the following approximate expressions developed through a simplified strength of materials approach:
composite constituent materials are: equilibrium and establish the strain compatibility between the respectively, as follows:

\[
\varepsilon' E_1^i = \frac{f'}{E_1} \left( 1 - f_d \right) + (1 - f') \left( 1 - m_d \right)^m E
\]

\[
\varepsilon' E_2^i = \frac{1}{\sqrt{f'}} \frac{\varepsilon' G_{12}^i}{(1 - f_d) (1 - m_d)^m E} + \frac{1 - \sqrt{f'}}{1 - m_d} \varepsilon' G_{12}^i
\]

\[
\varepsilon G_{12}^i = \frac{1}{\sqrt{f'}} \frac{\varepsilon G_{12}}{(1 - f_d) (1 - m_d)^m G} + \frac{1 - \sqrt{f'}}{1 - m_d} \varepsilon G_{12}
\]

\[
\varepsilon' G_{23}^i = \frac{1}{\sqrt{f'}} \frac{\varepsilon G_{23}^i}{(1 - f_d) (1 - m_d)^m G} + \frac{1 - \sqrt{f'}}{1 - m_d} \varepsilon G_{23}
\]

\[
\varepsilon' v_{12} = f' f' v_{12} + (1 - f') m v
\]

where the superscripts \( c, f \) and \( m \) stands for composite, fibre and matrix, respectively, \( E_i \) are the elastic moduli in the \( i \)th direction, \( v_{ij} \) the Poisson’s ratios, \( G_{ij} \) the shear moduli and \( f' \) is the fibre volume-fraction coefficient. The superscripts \( d \) indicate the effective ply degraded modulus affected by the scalar degradation parameters.

For a lamina that behaves as a linear orthotropic solid, the strains and stresses relationship is given by the effective compliance matrix \( S' \) expressed by

\[
\begin{pmatrix}
\varepsilon' E_1 \\
\varepsilon'E_2 \\
\varepsilon'E_3 \\
\varepsilon'\gamma_{13} \\
\varepsilon'\gamma_{23}
\end{pmatrix} =
\begin{pmatrix}
\frac{1}{E_1} & \frac{-\nu_{xy}}{E_2} & \frac{-\nu_{yx}}{E_2} & 0 & 0 & 0 \\
\frac{-\nu_{xy}}{E_2} & \frac{1}{E_2} & \frac{-\nu_{xz}}{E_2} & 0 & 0 & 0 \\
\frac{-\nu_{yx}}{E_2} & \frac{-\nu_{xy}}{E_2} & \frac{1}{E_2} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{G_{12}} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{G_{23}} & 0 \\
0 & 0 & 0 & 0 & 0 & \frac{1}{G_{12}}
\end{pmatrix}
\begin{pmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\tau_{13} \\
\tau_{23}
\end{pmatrix}
\]

where the ply is considered to be transversely isotropic, correspondingly: \( \varepsilon' E_3 = \varepsilon' E_2, \varepsilon G_{13} = \varepsilon G_{12}, \varepsilon' v_{13} = \varepsilon' v_{12} \) and

\[
\varepsilon' v_{23} = \frac{\varepsilon E_{23}^d}{2 \varepsilon G_{23}^d} - 1
\]

It should be noticed that the scalar damage model used does not account for directionality of damage. Nonetheless anisotropic damage effects on the macroscale are taken into account by means of the stacking sequence of the laminate. For the computation of the damage map prediction in the laminated structure, the general dynamic differential equation was solved by an explicit time integration scheme implemented in the explicit FE code ComPack, wherein the stiffness matrix is not assembled [14]. Only the mass, the nodal displacements, velocities and accelerations vectors are involved. However, to determine the eigenfrequencies and eigenvectors of the system, the mass \( M \) and stiffness \( K \) assembled matrices are necessary to solve the eigenvalue equation, expressed as:

\[
\Omega \Phi = K \Phi
\]

where \( \Omega = \text{diag} \left[ \omega_1^2 \cdots \omega_n^2 \right] \) is the diagonal matrix of eigenfrequencies and \( \Phi = [\varphi_1 \cdots \varphi_n] \) are the corresponding eigenvectors. Therefore, a specifically devised FE Matlab code was used to solve the eigenvalue (modal) analysis problem, in which the predicted map of damage in terms of internal
variables \( \delta d \) and \( \delta^m d \) was introduced on Gauss points of each finite element. The mass and stiffness element matrices are given by:

\[
\mathbf{M}' = \int_V \rho \mathbf{N}^T \mathbf{N} \, dV
\]

\[
\mathbf{K}' = \int_V \mathbf{B}^T \mathbf{C}(\delta d, \delta^m d) \mathbf{B} \, dV
\]

being and \( \mathbf{N} \) and \( \mathbf{B} \) are respectively the shape functions and strain matrices for the element.

3 NUMERICAL ANALYSIS

The numerical study conducted to analyse the vibration response of impact damaged laminates was undertaken in two stages. In the first stage, a prediction map of the impact-induced damage was obtained over a range of incident energies using the explicit FE code ComPack. In the second stage, the eigenvalues and eigenvectors are calculated using the specifically devised FE Matlab code for three different damage approaches on the different length scales.

3.1 Material

The monolithic quasi-isotropic composite specimen modeled was composed of 40 unidirectional CFRP plies with a balanced and symmetric stacking sequence \([45/0/-45/90]_3S\). The laminated plate orientation was defined such that the 0° fibre orientation was aligned with the lengthwise dimension. The size of the plate sample was \(150 \times 100 \times 5.2 \) mm\(^3\). The component mechanical properties and calculated homogenised ply mechanical properties are listed in Table 1. Transverse constituent properties given in [15] are used throughout this work to complete data provided by manufacturers.

<table>
<thead>
<tr>
<th>Fibre properties</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td>Grafilt TR30S 3K</td>
</tr>
<tr>
<td>Density</td>
<td>( \rho ) 1790 kg/m(^3)</td>
</tr>
<tr>
<td>Longitudinal modulus</td>
<td>( E_1 ) 235 GPa</td>
</tr>
<tr>
<td>Transverse modulus</td>
<td>( E_2 ) 15 GPa</td>
</tr>
<tr>
<td>In-plane shear modulus</td>
<td>( G_{12} ) 15 GPa</td>
</tr>
<tr>
<td>Major Poisson’s ratio</td>
<td>( v_{12} ) 0.21</td>
</tr>
<tr>
<td>Transverse shear modulus</td>
<td>( G_{23} ) 7 GPa</td>
</tr>
<tr>
<td>Tensile strength</td>
<td>( \sigma_t ) 4410 MPa</td>
</tr>
<tr>
<td>Elongation at failure</td>
<td>( E_L ) 1.9 %</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Matrix properties</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Type</td>
<td>HSC Epikote 4652 epoxy</td>
</tr>
<tr>
<td>Density</td>
<td>( \rho ) 1200 kg/m(^3)</td>
</tr>
<tr>
<td>Modulus</td>
<td>( E ) 4520 MPa</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>( v ) 0.358</td>
</tr>
<tr>
<td>Tensile strength</td>
<td>( \sigma_t ) 68 MPa</td>
</tr>
<tr>
<td>Fracture energy</td>
<td>( R ) 780 J/m(^2)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>UD lamina properties</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Fibre volume fraction</td>
<td>( \phi ) 0.552</td>
</tr>
<tr>
<td>Cured ply thickness</td>
<td>( h ) 0.130 mm</td>
</tr>
<tr>
<td>Weight</td>
<td>( w ) 200 g/m(^2)</td>
</tr>
<tr>
<td>Longitudinal modulus</td>
<td>( E_1 ) 131.66 GPa</td>
</tr>
<tr>
<td>Transverse modulus</td>
<td>( E_2 ) 8.53 GPa</td>
</tr>
<tr>
<td>In-plane shear modulus</td>
<td>( G_{12} ) 4.57 GPa</td>
</tr>
<tr>
<td>Transverse shear modulus</td>
<td>( G_{23} ) 3.26 GPa</td>
</tr>
<tr>
<td>Major Poisson’s ratio</td>
<td>( v_{12} ) 0.276</td>
</tr>
<tr>
<td>Through thickness Poisson’s ratio</td>
<td>( v_{23} ) 0.307</td>
</tr>
</tbody>
</table>

Nominal values provided by manufacturers, except those indicated by † which were adopted from [39] and ‡ from model prediction [36].

Table 1: Mechanical properties of the composite constituents and lamina.
3.2 Drop-weight impact simulation

The geometrical model depicted in Fig. 1 was built to reproduce the experimental setup described in the standard test method ASTM D7136. The virtual coupon consists of a 75×50×5.2 mm³ rectangular parallelepiped, which corresponds to one quarter of the overall geometry of the laminate. This simplification assumes a double symmetry in the internal damage distribution to significantly reduce the computational cost. The virtual coupon was simply supported on the base and was fixed preventing the movement in the out-of-plane direction during the impact of a hemispherical projectile. The impactor diameter was 16 mm, according to the standard test method. The fixture base and the impactor were modeled as solid rigid bodies. Only the holder and the composite virtual coupon were considered deformable parts.

![Numerical prediction of the through-thickness distribution of damage for a 50 J simulated impact on the composite virtual coupon (top). The variable plotted corresponds to the element average damage. Scheme of the serial-parallel continuum mechanics approach (bottom).](image)

The primary objective of this numerical simulation was to determine the map of induced damage and in particular the delamination failure. This is a inherently three-dimensional phenomenon. Accordingly, hexahedral continuum solid elements with eight nodes and full integration were used to model the composite coupon since all components of the strain and stress tensors are included in the formulation. As stated above, the use of the SP continuum mechanics approach has the chief advantage over competing approaches of not needing interface elements to capture the phenomenon of initiation and propagation of interlaminar damage. Consequently, the through-thickness discretization of the composite virtual coupon can be done by collapsing several layers into a single element, leading to a significantly decrease of the computational time required to perform the simulation. A finite element mesh of 60×40 with 12 elements in the through-thickness direction was selected to achieve an optimal performance, leading to a total of 32513 nodes. This procedure does not necessarily compromise the accuracy of the results as it is shown in [6, 11].

Numerical simulations were carried out for each level of the incident impact energy conducted on the experimental program as described in the next section. The impactor was modeled by defining the initial position of 1 mm onto the laminate and a vertical prescribed velocity consistent with the incident impact energy. Contacts were defined to set up the set of contact parts pairs, in order to avoid interpenetration between parts. The numerical results provided a map of induced damage in terms of internal variables $d$ and $\sigma d$ on each Gauss point of the finite element, which was employed in the subsequent stage to characterize the ply-group degraded behaviour.
3.3 Vibration response modeling

To determine the natural frequencies and modal vectors of the damaged laminated plate, the specifically devised FE simulation code was used to solve the eigenvalue (modal) analysis. A numerical modal analysis was performed under free boundary conditions in order to avoid uncertainties when compared with experimental tests. The properties of the composite virtual coupon are the same as that for the previous numerical study except for the geometric model, which corresponds to the whole plate dimension. The mesh discretization was kept identical with 12 elements in the through-thickness direction (see Fig. 1). This was a requirement to merge the internal damage data set. Three-dimensional multilayer hexahedral elements with eight nodes and full integration were used, collapsing several layers into a single element. The predicted map of induced damage in terms of internal variables $i_d$ and $\varepsilon_i$ was subsequently inserted in each homologous Gauss point, aiming to locally degrade the element constitutive behaviour.

In addition to the micro-mechanical approach described above, two additional modeling strategies were used to investigate the effect of impact damage upon the vibration response. The second approach is based on a homogenised damage model at the mesoscale level, which has been used by different researchers to model the mechanical performance of composite lamina taking into account the different failure mechanisms $[5, 16]$. Damage is characterised through the damage array $d_i = \{d_i\}$ for $i = 1...6$ where $d_i \in [0, 1)$, whose components provide the influence of each damage mechanism on the effective compliance (see Eq. 20) coefficients as:

$$S_{ij}^d = \frac{1}{(1 - d_i)} S_{ij} \quad \text{for} \quad i = j$$  \hspace{1cm} (20)

The dominant damage mechanisms are found in the laminate plane. When only delamination damage is taken into account, some authors suggest a simplified approach where only the transverse shear moduli are affected $[5]$. In that case, the damage factors $d_t$ and $d_i$ are chosen to describe the degradation of the transverse shear moduli $G_{13}$ and $G_{23}$, respectively. In the present work the lower and upper bounds of both mesoscale damage parameters were set to 0.84 and 0.94, respectively, similarly to previously reported works $[5]$. In order to compare the suitability and accuracy of this mesoscale approach, the same predicted map of induced damage was used to locally degrade the element constitutive behaviour, whereby the damaged extension was equivalent in both cases.

The third approach consists in physically subdividing the damaged virtual coupon by duplicating and detaching the nodes at the interface of adjacent elements at the location of the predicted delaminations. This can be similarly addressed by inserting a slim virtually degraded layer element between the upper and lower sublaminates within the delaminated regions. This strategy has been widely used previously $[5]$. In this work, slim interface solid elements were automatically created over the entire mesh. Then, the pristine constitutive models corresponding to the locations along the laminate thickness were assigned. Finally, at the location of predicted damaged elements the constitutive model of the interface elements was severely degraded (up to 1%) to account for the discontinuity conditions. The influence of the interface element thickness was investigated comparing results of 2.50% and 25.0% of the nominal element thickness. As in the above cases, the damaged extent was equivalent thus simplifying comparisons.

4 RESULTS AND DISCUSSION

The capability of the three different scale-modeling strategies used to investigate the effect of impact damage upon the vibration response are compared to experimental results in Figs. 2 and 3. The experimental data plotted represent mean values of the percentage frequency changes with standard deviation bars for each impact energy level. The shaded area indicates presence of fibre breakage. Numerical results correspond to the predictions of the different modelling strategies such as the addressed micro-mechanical approach by decoupling also the effect of fibre damage, the mesoscale approach based on a degradation of the transverse shear moduli $G_{13}$ and $G_{23}$, as well as the approach
consisting in inserting a slim virtually degraded layer elements between the upper and lower sublaminates within the interface of the delaminated regions, as previously explained.

In general terms, numerical simulations reproduce a variation of eigenfrequencies as impact energy increases. Nonetheless, markedly different trends between experimental results and numerical predictions are observed. While the micro-mechanical approach yields a reasonably accurate estimation over all impact energy ranges, mesoscale and interface approaches predictions significantly depend on the damage or interface parameter settings, respectively. For instance, settings dependency is highlighted in modes 6th, 8th, 9th, 10th or 11th, where frequency shift predictions vary considerably for the set lower and upper bounds.

It is worth noting that both the mesoscale and the interface approaches are aimed to represent solely the effect of interply decohesion. Accordingly, a direct dependency exists with the delaminated area. Regardless, other modes visibly distance themselves from this dependence thus underlining the role of fibre breakage. As a first approximation and in order to investigate the significance of fibre damage on the vibration response, eigenfrequencies shifts were calculated by cancelling the effect of damage on fibres, i.e. $d=0$. Those results correspond to the represented undamaged fibres curves, showing noticeable differences after the fibre damage onset. It should be noted that this hypothetical condition corresponds to an unrealistic situation, since if fibres are not considered to degrade, other failure mechanisms must dissipate the system energy during the impact event, thus leading to an increase of the matrix damaged area that it is not being considered here. Furthermore, as observed in Fig. 3, fibre degradation has a significant effect upon the in-plane modes. Results reveal that the micro-mechanical approach allowed us to reasonably reproduce this experimentally observed behaviour.

5 CONCLUSIONS

A FE modelling approach based on a micro-mechanical description of low-velocity impact-induced damage on CFRP laminated plates was investigated to analyse its effect upon the vibration response. The map of internal induced damage was firstly determined through the previously developed serial-parallel (SP) continuum approach [11, 12], wherein two compatibility equations define the stress equilibrium and establish the strain compatibility between the composite and the constituent materials. Subsequently, eigenvalue analysis was addressed to determine the modal response of impacted laminates. The validity of the modelling approach to successfully reproduce the vibration response of impacted coupons was assessed through a comparison with experimental measurements, showing a good correlation over all impact energy range evaluated.

Among the findings drawn from this study, it is shown that fibre breakage exerts a significant influence upon the in-plane vibration modes. On the other hand, the micro-mechanical approach overcomes several drawbacks of the other examined methods. Thus, for example, the abrupt and drastic global reduction of dominant stiffness components by using a homogenised scalar value on the mesoscale approach has been shown to be physically unrealistic, since this assumption is comparable to assuming an ideally brittle post-failure behaviour. Through the present micro-mechanical approach the progressive degradation of the constituent materials can be accounted for, thus providing a more realistic representation of the internal damage. Furthermore, by means of the interface-based approach, the insertion of slim virtually degraded layer elements within the delaminated regions entails an increase in the number of elements and, consequently, the computational cost is increased. On the micro-mechanical approach, intraply and interply damages are captured within the same framework thus does not require the explicit inclusion of individual plies. Hence, several layers can be collapsed into a single element, which results in a computationally less demanding formulation.

In summary, the present micro-mechanical modeling approach is shown to be a promising numerical tool to be used in model-based vibration methods for impact damage identification routines. Studies to investigate the capability and suitability for modelling higher impact regimes on large structures will be the subject of future research.
Figure 2: Experimental and numerical predictions of the percentage frequency changes for different modeling approaches as a function of incident kinetic energy of the first 13 transverse modes and first 3 in-plane modes. IDO refers to the interlaminar damage onset. (Follow in Fig. 3.)
Figure 3: Experimental and numerical predictions of the percentage frequency changes for different modeling approaches as a function of incident kinetic energy of the first 13 transverse modes and first 3 in-plane modes. IDO refers to the interlaminar damage onset. (Comes from Fig. 2.)
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