

RECENT DEVELOPMENTS OF DISCRETE MATERIAL OPTIMIZATION OF LAMINATED COMPOSITE STRUCTURES

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ABSTRACT

This work will give a quick summary of recent developments of the Discrete Material Optimization approach for structural optimization of laminated composite structures. This approach can be seen as a multi-material topology optimization approach for selecting the best ply material and number of plies in a laminated composite structure. The conceptual combinatorial design problem is relaxed to a continuous problem such that well-established gradient based optimization techniques can be applied, and the optimization problem is solved on basis of interpolation schemes with penalization. The different interpolation schemes used are described, and it is briefly outlined how design rules/manufacturing constraints can be included in the optimization. The approach has been demonstrated for a number of global design criteria like mass, compliance, buckling load factors, etc., but recent work makes it possible also to include local criteria such as strength criteria in the formulations. This is illustrated by structural optimization of a corner hinged laminated plate in this paper, and at ICCM20 it will also be demonstrated for optimization of a main spar from a wind turbine blade.

1 INTRODUCTION TO DISCRETE MATERIAL OPTIMIZATION

The so-called Discrete Material Optimization (DMO) approach developed by Stegmann and Lund [1-2] is a parameterization method that can be used to solve the fiber orientational problem of monolithic laminated composites, where a number of discrete fiber angles are allowed. It can also be used for designing hybrid composites or sandwich structures where the orientational problem together with the right choice of material is to be solved. A number of candidate materials are proposed which could be a fiber reinforced polymer (FRP) material oriented at different chosen fiber angles together with possible core materials in case of designing sandwich structures. The discrete optimization problem is then converted to a continuous problem using interpolation functions with penalization, such that it is possible to apply efficient gradient based optimization algorithms for solving the multi-material optimization problem. The approach can thus be seen as an extension of multiphase topology optimization of Sigmund and co-workers [3-4].

The laminated composite structure is modeled by layered shell finite elements, and the structure is divided into a number of patches, consisting of a number of finite elements, where the same layup should apply. A number of candidate materials, n^c , are defined for each material patch p , and the candidate material variables, x_{plc} , are defined for all n^p material patches such that

$$x_{plc} = \begin{cases} 1 & \text{if candidate } c \text{ is selected in layer } l \text{ of patch } p \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

The constitutive matrix \mathbf{E}_{el} for a given layer l in a given shell element e associated with patch p is thus determined by

$$\begin{aligned}\mathbf{E}_{el} &= \sum_{c=1}^{n^c} x_{plc} \mathbf{E}_c \\ \sum_{c=1}^{n^c} x_{plc} &= 1 \quad \forall(p, l) \\ x_{plc} &\in \{0; 1\} \quad \forall(p, l, c)\end{aligned}\tag{2}$$

where \mathbf{E}_c is the constitutive matrix associated with material candidate c . Next, the combinatorial problem of selecting the material candidate variables x_{plc} is converted to a continuous problem using interpolation functions with penalization.

1.1 The original DMO schemes 4 and 5

In the first DMO approach by Stegmann and Lund [1-2] two different interpolation schemes were most successful in performance, and they have been denoted DMO scheme 4 and 5 according to their numbering in [1]. In DMO scheme 4 the weight functions \hat{w}_{plc} are given by

$$\hat{w}_{plc} = (x_{plc})^p \prod_{j=1; j \neq c}^{n^c} (1 - (x_{plj})^p), \quad 0 < \underline{x}_{plc} \leq x_{plc} \leq \overline{x}_{plc} < 1\tag{3}$$

Here p is a penalization power that typically is set to 1 initially and increased during the optimization to 3 or 4 in order to penalize intermediate values of design variables. The lower and upper bounds on design variables are set close to 0 and 1, respectively. However, these self-balancing weight functions for scheme 4 might not add to unity, and therefore the normalized version, DMO scheme 5, was introduced by defining the weight functions w_{plc} as

$$w_{plc} = \frac{\hat{w}_{plc}}{\sum_{k=1}^{n^c} \hat{w}_{plk}} \quad \forall(p, l, c)\tag{4}$$

A constitutive matrix, or any other material parameter, is thus interpolated by the following expression for a given layer l in a given element e

$$\mathbf{E}_{el} = \mathbf{E}_0 + \sum_{c=1}^{n^c} w_{plc} \Delta \mathbf{E}_c\tag{5}$$

where $\Delta \mathbf{E}_c = \mathbf{E}_c - \mathbf{E}_0$ and \mathbf{E}_0 is the constitutive matrix of a weak material, such that $\Delta \mathbf{E}_c$ is positive definite. The DMO approach using schemes 4 and 5 has been demonstrated for a number of applications involving compliance, mass, eigenfrequencies, buckling load factors, vibro-acoustics, and multi-criteria design optimization problems, see e.g. [1-2,5-6]. However, in several cases these interpolation schemes have difficulties in obtaining a distinct choice of material, and therefore several other interpolation schemes have been suggested.

1.2 Multiphase SIMP and RAMP schemes

Alternatively to the self-balancing procedure used for scheme 4 and 5, Hvejsel et al [7] introduced a series of linear equality constraints to ensure that the sum of weighting functions for the candidate materials would equal unity, whereas the distinct selection of a single candidate was achieved by a non-linear inequality constraint. This work was extended in Hvejsel and Lund [8] where it was proposed to use multi-material variations of the well-known SIMP and RAMP interpolation schemes from topology optimization with isotropic materials, see [9] and [10]. In the multiphase SIMP scheme a constitutive matrix is computed as

$$\mathbf{E}_{el} = \mathbf{E}_0 + \sum_{c=1}^{n^c} \underbrace{x_{plc}^p}_{=w_{plc}} \Delta \mathbf{E}_c, \quad 0 \leq x_{plc} \leq 1, \quad \sum_{c=1}^{n^c} x_{plc} = 1 \quad \forall(p, l)\tag{6}$$

Thus, the design variables x_{plc} can be considered as volume fractions of candidate materials. The penalization power p is used in a similar way as for scheme 4 and 5 to penalize intermediate values of design variables. In a similar way the multiphase RAMP scheme is defined as

$$\mathbf{E}_{el} = \mathbf{E}_0 + \sum_{c=1}^{n^c} \underbrace{\frac{x_{plc}}{1 + q(1 - x_{plc})}}_{=w_{plc}} \Delta \mathbf{E}_c, \quad 0 \leq x_{plc} \leq 1, \quad \sum_{c=1}^{n^c} x_{plc} = 1 \quad \forall(p, l) \quad (7)$$

The penalization parameter q plays the same role as p in the multiphase SIMP scheme. It is typically set to 0 initially, such that no penalization is enforced initially, and it is then increased during the optimization. The main advantage of the RAMP scheme is that it has a nonzero gradient for $x_{plc} = 0$.

The multiphase SIMP and RAMP schemes lead to very many sparse linear constraints due to the resource constraint that the sum of the material design variables should be 1. Thus, it is necessary to use optimization algorithms which can handle such linear constraints efficiently. An example is the CPLEX optimizer by IBM ILOG [11] which is our preferred optimizer for these problems.

1.3 Other DMO schemes

Kennedy and Martins [12] presented another approach for obtaining discrete designs. Here, the authors suggested to apply the multiphase SIMP interpolation scheme without penalization, and instead they added a series of non-linear equality constraints as a penalty term to the objective function, effectively penalizing intermediate valued design variables.

As an alternative to the DMO schemes described above, Bruyneel [13] proposed the Shape Functions with Penalization (SFP) method. Here, the author applied four node shape functions known from the finite element method to interpolate between four material candidates using only two design variables. This is in contrast to the DMO method which requires one design variable for each material candidate. The SFP method was later extended to include three and eight node elements, see [14]. Later, Gao et al [15] generalized the SFP method by introducing the Bi-valued Coding Parameterization (BCP) method. Compared to SFP, this method has no upper limit on the number of applied material candidates while still providing a substantial reduction in the number of design variables required to do the material interpolation.

2 DMTO – VARYING THICKNESS LAMINATES

The DMO approach was extended to varying thickness laminates in [16-17] where the Discrete Material and Thickness Optimization (DMTO) approach was developed, making it possible to simultaneously determine an optimum thickness variation and material distribution of the laminated composite structure. The idea is to introduce a density variable to govern the presence of material in a given layer, and thereby determine the thickness variation throughout the laminate. The layer-wise density variables can be defined either on element level or by groups. They are defined on element level for all n^e elements such that

$$\tilde{\rho}_{el} = \begin{cases} 1 & \text{if there is material in layer } l \text{ for element } e \\ 0 & \text{otherwise} \end{cases} \quad (8)$$

In a similar way as described for the material design variables, the density variables $\tilde{\rho}_{el}$ are treated as continuous variables and the constitutive properties for a given layer in a given element can now be interpolated using a SIMP scheme approach as

$$\mathbf{E}_{el} = \mathbf{E}_0 + \tilde{\rho}_{el}^q \sum_{c=1}^{n^c} x_{plc}^p \Delta \mathbf{E}_c$$

$$\tilde{\rho}_{el} \in [0; 1] \quad \forall(e, l) \quad (9)$$

$$\sum_{c=1}^{n^c} x_{plc} = 1 \quad \forall(p, l)$$

Here q is a penalization power for the density variables $\tilde{\rho}_{el}$. The resulting parameterization is non-

convex, even for $p = q = 1$, and therefore the solutions obtained are typically local optima. In Sørensen and Lund [16] the authors developed an explicit manufacturing constraint for preventing intermediate void to appear in the laminated structure, but by making use of filters known from traditional topology optimization, an alternative DMTO approach has recently been proposed by Sørensen and Lund [18]. The readers are referred to these papers for details regarding the implementation.

3 MANUFACTURING CONSTRAINTS

In the DMO and DMTO formulations it is possible to include design rules/manufacturing constraints. These are typically experience based rules that reduce the risk of failure modes associated to out-of-plane stresses which are not captured very accurately by the layered shell elements used for the analysis. Many of these constraints have their origins in the aerospace industry, see e.g. Kassapoglou [19], and examples include contiguity constraints (limits on the number of identical plies), adjacency or so-called blending rules, allowable thickness changes (ply drop rules), and requirements for symmetric and balanced layups. As the DMO and DMTO formulations have their origin in the discrete problem to be solved, it is possible to formulate these constraints in their discrete form and include them in their relaxed, continuous form when solving the optimization problem. A detailed description of this is given in Sørensen and Lund [16], see also [17-18].

4 STRENGTH CRITERIA

Most of the DMO and DMTO papers have focused on design optimization problems including global criteria such as mass, compliance, buckling load factors, and eigenfrequencies. Local criteria like displacement constraints can also easily be included, see e.g. [17-18], whereas it is more challenging to include strength criteria. The DMO/DMTO formulations result in very many design variables, and including strength criteria typically result in very many nonlinear constraints. Thus, a feasible engineering solution to this large scale problem is to group the many strength values into a number of global strength measures using aggregation functions like p-norm functions or Kreisselmeier-Steinhauser functions. Such formulations have successfully been applied in structural topology optimization using isotropic materials, see e.g. [20-21], and similar formulations can be applied for multi-material problems using DMO/DMTO as demonstrated in Lund et al [22] where more details are provided.

5 NUMERICAL EXAMPLE: 8-LAYER CORNER HINGED PLATE

At ICCM20 an example of multi-criteria design optimization of a main spar from a wind turbine blade will be shown, demonstrating the use of the DMTO approach including both global criteria and local strength criteria. In this paper a benchmark example is included to illustrate the DMTO approach for minimizing the maximum failure index of an 8-layer corned hinged plate subjected to a mass constraint. The dimensions of the plate are given in Fig. 1 together with the boundary conditions of the static problem.

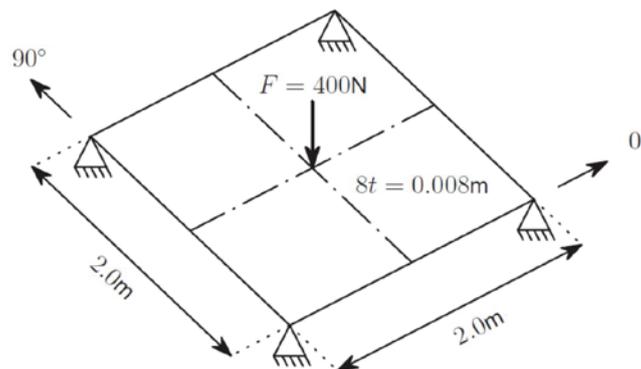


Figure 1: Definition of 8-layer corned hinged plate example, where t denotes the layer thickness.

The 8-layer plate is made of a glass fiber reinforced polymer (GFRP), see elastic constraints and allowable strain values in Table 1.

Property		Value
E_1	[GPa]	34.0
E_2	[GPa]	8.2
ν_{12}	–	0.29
G_{12}	[GPa]	4.5
G_{13}	[GPa]	4.5
G_{23}	[GPa]	4.0
ρ	[kg/m ³]	1910
Longitudinal tensile failure strain ϵ_{1t}	[%]	2.45
Longitudinal compressive failure strain ϵ_{1c}	[%]	1.5
Transverse tensile failure strain ϵ_{2t}	[%]	0.37
Transverse compressive failure strain ϵ_{2c}	[%]	1.22
In-plane (12) failure shear strain γ_{12u}	[%]	1.94
Transverse (13) failure shear strain γ_{13u}	[%]	1.94
Transverse (23) failure shear strain γ_{23u}	[%]	1.2

Table 1: Elastic properties and strength parameters for GFRP.

It is assumed that the strength can be estimated using the maximum strain criterion, and the objective of the optimization problem is to minimize the maximum failure index, subject to the constraint that only half of the domain can be filled with material. The plate is modelled using a 48 x 48 mesh of 9-node isoparametric shell elements. The parameterization of material distribution is such that each layer must have the same GFRP material, and the allowable fiber angles are 0°, ±45°, or 90°. Contiguity constraints are applied, such that a maximum of 4 identical layers is allowed. The thickness of the plate is allowed to vary, and for the thickness parameterization a 24 x 24 patch description is used (with 2 x 2 shell elements in each patch). A slope constraint (ply drop constraint) is applied, stating that a maximum drop of 1 layer is allowed between each thickness patch.

The failure indices computed using the maximum strain criterion are evaluated in the center of each shell element at both bottom and top of each ply, resulting in 36,864 potential failure indices to minimize. The number of failure indices is reduced to 1 for each thickness patch using a p-norm aggregation function, and the min-max problem of minimizing the maximum failure index, subject to the mass constraint, is solved using a bound formulation, see [23]. The number of design variables is 1,041.

The material of the initial design is a mixture of the 4 candidate materials (equal weighting), i.e. a non-physical material, and it should be noted that the initial design is infeasible, as the mass is twice the allowable value. The maximum strain failure index of the initial design is given in Fig. 2.

The choice of fiber angle and thickness distribution of the final design is seen in Fig. 4. The solution is a 0/90 cross ply laminate with varying thickness, and all the constraints are satisfied.

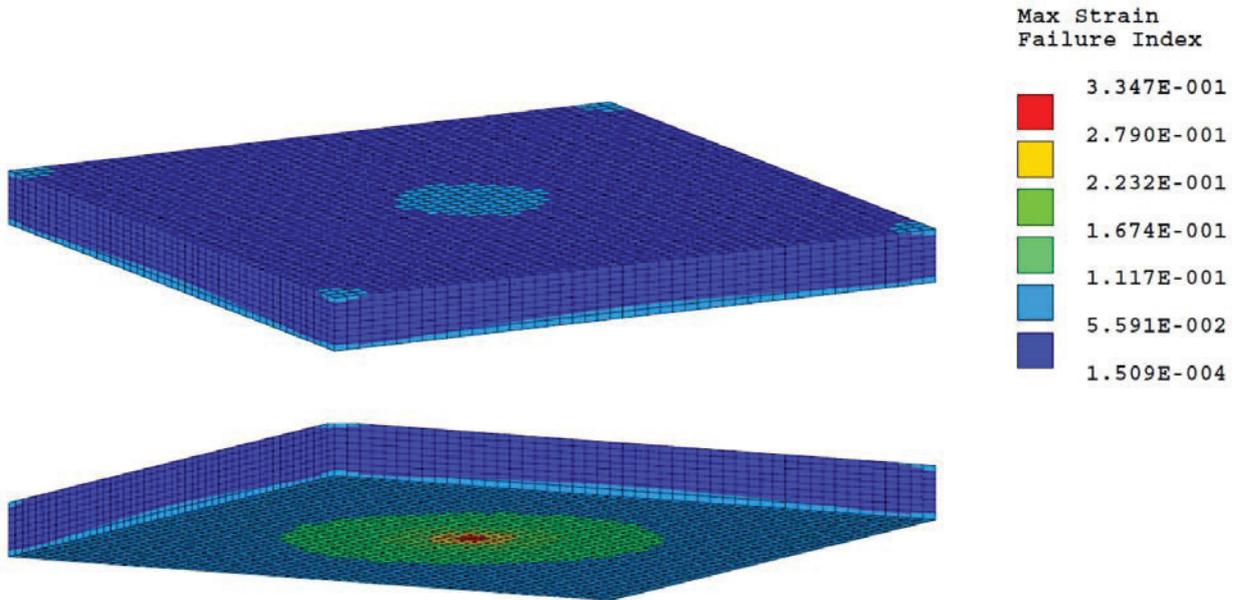


Figure 2: The maximum strain failure index of the initial design is illustrated using a solid model where thicknesses are scaled by a factor of 20.

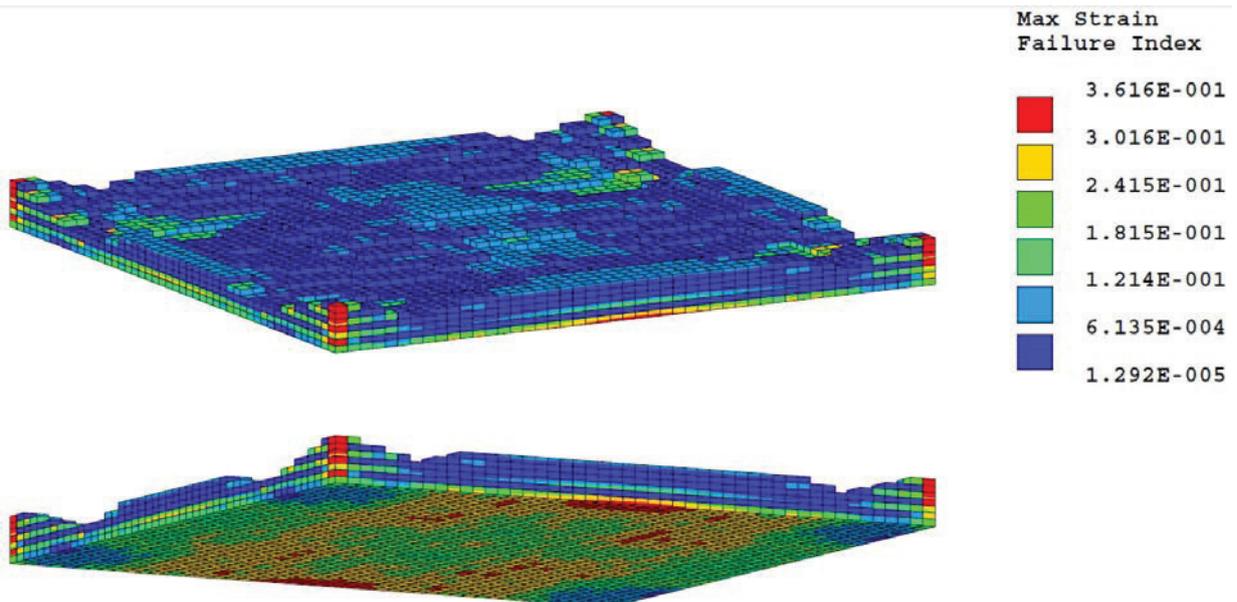


Figure 3: The maximum strain failure index of the optimized design.

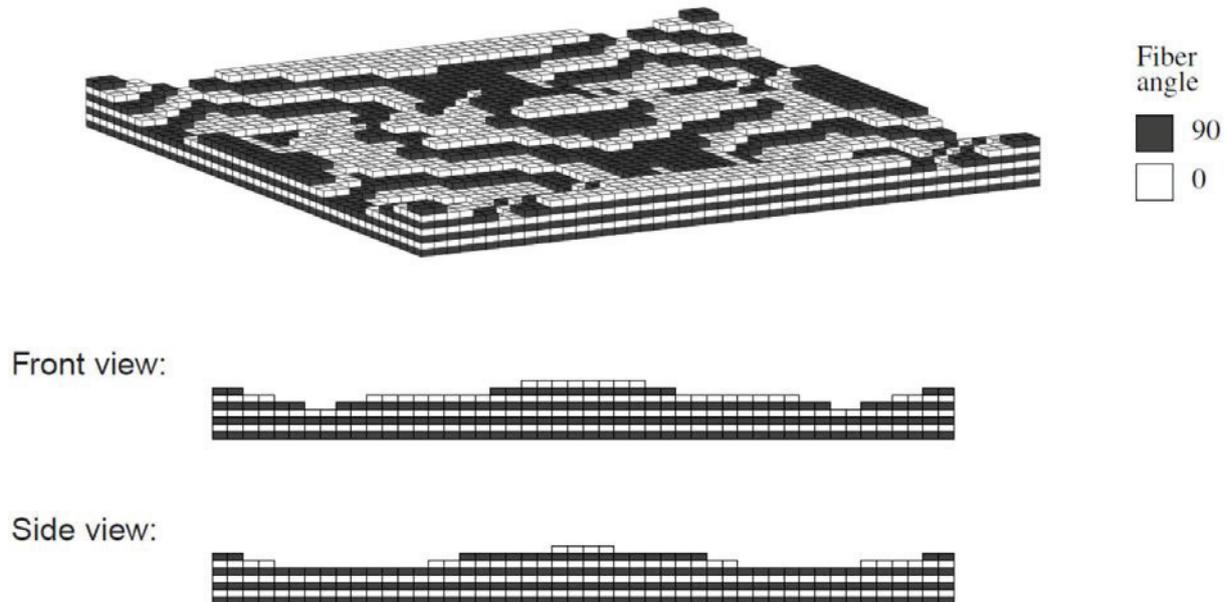


Figure 4: The layout of the optimized design.

The solution is not symmetric in-plane due to the requirement of having the same GFRP material in each layer. It is seen in Fig. 3 that a nearly uniform distribution of failure index is obtained in the bottom layer of the plate and in the corners where the plate is hinged. The front and side views given in Fig. 4 illustrate a small asymmetry which is to be expected when an arbitrary mass constraint is applied. The penalizations used in the interpolation schemes enforce a 0/1 design where the mass constraint is active, and with a given parameterization this might not result in a completely symmetric solution.

6 CONCLUDING REMARKS

The paper has given a quick summary of some recent developments of the Discrete Material Optimization approach including its extensions to solve discrete material and thickness distribution problems including local and global structural criteria together with design rules/manufacturing constraints. The approach results in very many design variables, however, modern optimization algorithms are capable of solving such large scale problems very efficiently. The DMO and DMTO approaches thereby offer designers of laminated composite structures the possibility of obtaining a very good preliminary design where structural criteria like stiffness, mass, strength, buckling load factors, eigenfrequencies, etc., can be taken into account, together with design rules/manufacturing constraints. The finite element analysis models are based on layered shell elements which capture global properties accurately whereas stresses and strains are only captured in an average sense, in particular in case of varying thickness laminates. The proposed DMTO parameterization for simultaneous determination of thickness variation and material distribution results in exterior ply drops, and thus the designs obtained need some modification as interior ply drops are normally preferred. Thus, the designs obtained should be considered as very good preliminary designs that only need few modifications in the final design stage.

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